LETTER TO THE EDITOR

A Comment on Nonlinear Analysis

Dear Sir:

We have been informed that Dr. Michael Korenberg (1973) had obtained a result similar to the formula we derived for the second-order frequency response of a linear-nonlinear-linear sandwich model (Victor and Shapley, 1980). Dr. Korenberg's work is now readily available in the book by Marmarelis and Marmarelis (1978). Dr. Korenberg's result applies to a linear-nonlinear-linear sandwich in which the nonlinearity is a power series or describable to an arbitrary degree of accuracy by a polynomial. He calculates the second-order cross correlation of such a system's output to a Gaussian white noise input. He shows that the Fourier transform of the second-order cross correlation, $\phi_{yxx}(\omega_1, \omega_2)$ in his terminology, is

$$\phi_{yxx}(\omega_1, \omega_2) = A_2 K(\omega_1 + \omega_2) G(\omega_1) G(\omega_2) \tag{1}$$

or in the time domain

$$\phi_{yxx}(\sigma_1, \sigma_2) = A_2 \int_0^\infty k(v) g(\sigma_1 - v) g(\sigma_2 - v) \mathrm{d}v \tag{2}$$

where $G(\omega)$ is the transfer function of the filter before the nonlinearity of the sandwich and $K(\omega)$ is the transfer function of the filter which follows the nonlinearity. A_2 is a constant which depends on the nature of the nonlinearity.

The expression for the second-order frequency response of a linear-nonlinear-linear sandwich in our Appendix B (Victor and Shapley, 1980) is quite similar in application and identical in algebraic form to Korenberg's first result above. The expression in our Appendix B for the second-order Wiener kernel for this model is likewise similar to Eq. 2 above, derived by Korenberg for the second-order cross correlation. Our derivation is different because it requires that the nonlinearity be approximable by an infinite series of Hermite polynomials. Moreover, we explicitly cast the result in the framework of orthogonal functional analysis. Our application of this equation to a system's response to a sum of sinusoids is, so far, unique.

The reference we made to Korenberg's other work in our article (Victor and Shapley, 1980 p. 471) may perhaps be of more significance to readers of the Biophysical Journal. In his 1973 paper, Korenberg showed how to calculate the Volterra kernels of linear-nonlinear-linear-nonlinear-linear sandwiches. While the range of application of Volterra theory is narrower than Wiener theory, this result of Korenberg's could be quite useful in the analysis of systems to which Volterra theory applies. No comparable result exists for the Wiener kernels or frequency kernels of linear-nonlinear-linear-nonlinear-linear sandwiches, to our knowledge. Therefore, at present the analysis of such systems is restricted to the Volterra theory in which the nonlinearities are analytic.

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